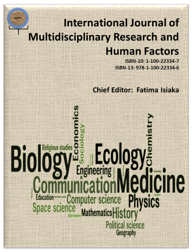

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Application of queuing theory on customer management in supermarket

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Abstract

The formation of a waiting line is a prevalent scenario when the immediate demand for a service surpasses the current capacity to provide that service. This study investigates the application of queuing theory on customer management in supermarkets using the Markov chain process across three consecutive months, represented by transition matrices. Each matrix outlines the probabilities of transitioning between four states $(0, 1, 2, 3)$ over time. The dataset was analysed using TORA software and the following estimates were obtained. The transition matrix for the first month reveals that state 0 is absorbing, while other states show varied probabilities of transitioning to other states. In the second month, state 1 consistently transitions to state 3 with a probability of 1.04, highlighting a significant shift in state dynamics. The third month's matrix continues to reflect these transitions, with state 0 remaining absorbing and other states exhibiting notable probabilities of either remaining in their current state or transitioning to others. These findings demonstrate the evolving nature of the system's state transitions over time, offering insights into the behaviour and stability of the Markov process.

Keywords : Service surpasses, Transition matrix, Markov process, Queuing theory, Customers time management, Markov chain model

1 Introduction

Queuing theory, also known as waiting line theory, is a mathematical study of waiting lines or queues (Odewale, (2016)). It is a statistical model that analyzes congestion and delays from waiting in line to receive a service or product. The concept of queuing theory has its origin in research that aimed to manage the flow of telephone calls and reduce waiting times in telephone exchanges. Today, it is applied to diverse fields, such as healthcare, transportation, and retail, to name a few. According to (Tian and Tong (2011)), the two key elements of queuing theory are the client's population source and the service system. The client's population source refers to the group of individuals or items waiting in line for service. The service system, on the other hand, is the process that serves the clients. Queuing theory considers various factors such as arrival rates, service rates, queue lengths, and wait times to predict and manage queues. Wait times are essential in almost every economic activity, and queuing theory can provide insights into optimizing resources and improving service quality. For instance, a grocery store with limited cash registers may experience queues, and queuing theory can be used to predict queue lengths, waiting times, and the number of cash registers needed to reduce wait times.

2 Literature Review

Queuing theory is a mathematical study of waiting lines or queues that uses a statistical model to analyze congestion and delays in receiving a service or product. It considers various factors such as arrival rates, service rates, queue lengths, and wait times to predict and manage queues. The concept of queuing theory is applied to diverse fields, and it provides insights into optimizing resources and improving service quality (Bereket (2016)). In the context of queuing theory applied to Nigerian supermarkets, let's consider the service system in many supermarkets. In this scenario, customers arrive at the supermarket and join the appropriate queue after shopping, equipped with relevant stationery. They patiently wait for the next available cashier to serve them, after which they promptly exit the system. In instances where no customers are waiting in the queue, the next customer is served immediately and then leaves the system. This process efficiently manages the flow of customers within the supermarket, optimizing the overall service experience.

Many works have been carried out on the queuing system and customers time management in (Kasum (2006)) worked on queue efficiency in Nigeria, (Pei-chun and Ann 2006) assessed service efficiency with application of queuing theory in Taiwan, while (Toshiba, Sanjay & Anil (2013) and Anichebe (2013)) worked on the application of queu-ing theory for the improvement in the queuing problem of, also (Williams, Ogege & Ideji (2014)) worked on effective customer service on bank profitability using queuing approach, (Obinwanne & Odunukwe (2015) and Ugawa, Okonkwo & Okonkwo (2015)) appraised queuing theory effectiveness in the management of customers time in, (Bereket (2016)) used queuing modelling for comparative study of in (Ethiopia and Odewale (2016)) investigated waiting lines, banks effective delivery and technology driven services in Nigeria.

In spite of several works on queuing model and customer time management in, considerable attention has not been given to types of queuing as it affect customers' time management. Furthermore, little research work has been done in comparing queuing systems in old-generation supermarkets to the new generation. Service delivery in most supermarkets is still inadequate. A long queue can still be seen at the halls and the ATM and POS service points, customers complain of waiting too long in halls with absolutely no rest of mind because of insecurity in the country and other service failures such as non-functioning of ATMs, POS, ATM debit without pay, non-availability of staff at service points, unprofessional conduct or rude behaviours by the staff of the poor standard of records or improper information, failed promises among others ((Idowu, Aliu & Adagunodo, 2002 ; Farabiyi, (2016)). (Kimes, S. E. (1989)), the study applied a Markov chain model to analyze customer arrival patterns and service rates in a supermarket setting. The Markov chain model allowed for more effective capacity management and pricing strategies by providing insights into customer arrival patterns and service rates. The Markov chain model proved to be a useful tool for optimizing capacity management and pricing in a supermarket setting. (Sainfort, F., Patiwoda, P. J., & Hanson, S. J. (1990)), The authors used a Markov chain model to optimize the number of open checkout lanes in a supermarket, taking into account customer waiting times and labour costs. The Markov chain model enabled the optimization of the number of open checkout lanes, balancing customer waiting times and labour costs. The Markov chain model was an effective approach for determining the optimal number of open checkout lanes in a supermarket environment.

(Korukoğlu, S., & Ballı, S. (2011)), this study incorporated a Markov chain model to predict customer transition probabilities between product categories in a supermarket. The Markov chain model provided insights into customer purchasing patterns, informing strategic product placement and inventory management. The Markov chain model was a valuable tool for forecasting customer transitions between product categories, which can guide supermarket merchandising and inventory decisions.

(Seiler S., Tuchman, A., & Yao, S. (2019)), The authors used a Markov chain model to analyze the effects of tax and subsidy policies on customer purchasing behavior in a supermarket setting. The Markov chain model offered insights into how tax and subsidy policies influence customer purchasing behavior in a supermarket environment. The Markov chain model proved to be a useful approach for evaluating the impact of tax and subsidy policies on customer purchasing patterns in a supermarket context, providing valuable information for policymakers.

(Breier, M., Falkenreck, C., & Wagner, G. (2019)), This study employed a Markov chain model to investigate the effectiveness of in-store promotional strategies in influencing customer purchase patterns within a supermarket. The Markov chain model enabled the researchers to understand how in-store promotional strategies impact customer purchase patterns in a supermarket setting. The Markov chain model was a successful tool for assessing the effectiveness of various in-store promotional strategies in shaping customer purchasing behavior within a supermarket environment.

(Chang, H. H., & Wen, F. H. (2011)), The authors integrated a Markov chain model with grey relational analysis to identify and optimize product bundling and cross-selling opportunities in a supermarket setting. The combined Markov chain and grey relational analysis approach provided insights into product bundling and cross-selling strategies that could be implemented in a supermarket to enhance sales and profitability. The integration of a Markov chain model and grey relational analysis was an effective way to uncover and optimize product bundling and cross-selling opportunities in a supermarket context. This study specifically aim to determine how the factors affecting waiting time in queue and to outline different strategies on reducing customers waiting time. Furthermore the remaining part of this work is structured as follows : section 2 covers the methods to be determined, section 3 analyse and discusses the results, and section 4 conclusion with the overall results

3 Materials and methods

3.1 Transition probability matrix

Consider a finite Markov Chain with r possible states, $x_1, x_2, \ldots, x_r$ Let p_{ij} be the conditional probability that the process will be in state x_j given that it was in state x_i at the preceding observation time. The transition probability matrix of the Markov Chain is defined to be the rxr matrix P with elements p_{ij}. Thus,

$$\begin{bmatrix} P_{11} & P_{12} & \ldots & P_{1r} \\ P_{21} & P_{22} & \ldots & P_{2r} \\ \ldots & \ldots & \ldots & \ldots \\ P_{31} & P_{32} & \ldots & P_{3r} \end{bmatrix} \tag{1}$$

Note that $P_{ij} \geq 0$ and $\forall i$ we have :

$$\begin{aligned} \sum_{j=1}^{N} P_{ij} &= \sum_{j=1}^{N} P_{ij} \left(X_{t+1} = \frac{j}{X_t} = i \right) \\ &= \sum_{j=1}^{N} P_{X_{t=1}} \left(X_{t+1} = j \right) = 1 \end{aligned} \tag{2}$$

where P is the transition probability matrix. The $(i,j)^{th}$ entry of P $\left(i^{th}\text{row and } j^{th}\text{columns is } P_{ij}\right)$. P is a square matrix.

Each row of P has entries that sum to 1. In other words, each row of p is a probability distribution over S (indeed the i^{th} row of P is the conditional distribution of X_n given that $X_{n-1} = 1$) for this reason P is said to be the stochastic matrix.

3.2 N-Step transition probability matrix

For any value of $n(n = 2, 3, \ldots)$, the n^{th} power of the matrix P in Equation 2 above which specify the probability that the chain will be in state j after $n - steps$ given that it begins in state i is called the n-step probability matrix. In general, the $n - step$ transition probability matrix

$$\begin{aligned} &P^{(n)} = P_n = P^{n-1}P, n \geq 1 \\ \\ &n = 1, P^{(1)} = P^0 P \\ &n = 2, P^{(2)} = P^1 P = P^0 P2 \\ &n = 3, P^{(3)} = P^2 P = P^0 P3 \\ &P^{(n)} = P^{(n-1)}, p = P^{(0)} P^{(n)} \\ &n = \text{how many time step in future.} \\ &ij = \text{going from state } i \text{to state } j \\ &P^{(n)} = n^{th} \text{power of transition propability} \\ &P_{ik}^{n-1} = \text{Result of the recursion from state} i \text{to} k \\ &\text{which is somewhere in between state } i \text{and} j \\ &\text{Markov chain recursion for} n - steps \text{transition} \end{aligned} \tag{3}$$

3.3 Steady state probability of a Markov chain

A characteristic of what is called a regular Markov chain is that, over a large enough number of iterations all transition probabilities will converge to a value and remain unchanged. This means that, after a sufficient number of iterations, the likelihood of ending up in any given state of the chain is the same regardless of where you start. When the chain reaches this point we say the transition probabilities reached a steady state. Consider a Markov Chain with r-states and the row vector

$$\pi = (\pi_1, \pi_2, \ldots, \pi_r) \tag{4}$$

such that :

- $\pi_1 \geq 0$
- $\sum_{i=1} \pi_1 = 1$
- $\pi_j = \lim_{n \to \infty} \pi_{ij}^n$

where

P_{ij} is defined above, then $(\pi_1, \pi_1, \ldots, \pi_r)$ is called the steady state vector of the Markov Chain. This means that as $n \to \infty$, the probability that the chain will transit from state x_i to a state $x_(j)$ is independent of the initial state x_i. π can be obtained by solving the relation $\pi = \pi p$ where $\pi =$ the initial state probability vector and $P =$ the state transition matirx.

4 Results and Discussion

This study considers the daily queuing activities used to select customers from within Crown Supermarket Kano for data collection. This ensures each customer in the population has an equal probability of being included in the sample. Customers who arrived at checkouts on sampling days were assigned sequential numbers. A random number generator then picked unique numbers to determine the 100 customers to be observed. This renders the sample representative of the overall customer population.

4.1 Steady-State Probabilities

4.2 First Month

Table 1 – The transition matrix for the first month.

States	0	1	2	3
0	0.0	0.000	0.000	0.000
1	0.0	0.500	0.250	0.250
2	0.0	0.333	0.500	0.167
3	0.0	0.000	0.333	0.656

From state 0 in the first month, the probability of remaining in state 0 in the second month is 1.0(100%). From state 1, there is a 50% chance (0.500) of staying in state 1, a 25% chance (0.250) of moving to state 2, and a 25% chance (0.250) of moving to state 3. From state 2, there is a 33.3% chance (0.333) of staying in state 2, a 50% chance (0.500) of moving to state 3, and a 16.7% chance (0.167) of moving to state 3. From state 3, there is a 33.3% chance (0.333) of moving to state 2 and a 65.6% chance (0.656) of staying in state 3.

4.3 Second Month

Table 2 – The transition matrix for the second month.

States	0	1	2	3
0	0.0	0.000	0.0	0.000
1	0.0	0.000	0.0	1.000
2	0.0	0.125	0.5	0.375
3	0.0	0.000	0.4	0.602

From state 0 in the second month, the probability of remaining in state 0 in the third month is 1.0(100%). From state 1, there is a 100% chance (1.000) of moving to state 3. From state 2, there is a 12.5% chance (0.125) of staying in state 2, a 50% chance (0.500) of moving to state 2, and a 37.5% chance (0.375) of moving to state 3. From state 3, there is a 40% chance (0.400) of moving to state 2 and a 60.2% chance (0.602) of staying in state 3.

4.4 Third Month

From state 0 in the third month, the probability of remaining in state 0 is 1.0(100%). From state 1, there is a 42.9% chance (0.429) of staying in state 1, a 42.9% chance (0.429) of moving to state 2, and a 14.3% chance (0.143) of moving to state 3. From state 2, there is a 33.3% chance (0.333) of staying in state 2, a 44.4% chance (0.444) of moving to state 2, and a 22.2% chance (0.222) of moving to state 3. From state 3, there is a 33.3% chance (0.333) of moving to state 2 and a 66.7% chance (0.667) of staying in state 3. These transition matrices show the probabilities of moving between different states over three months. For example, from the first month to the second month, there are clear transitions :

Table 3 – The transition matrix for the third month.

States	0	1	2	3
0	0.0	0.000	0.000	0.000
1	0.0	0.429	0.429	0.143
2	0.0	0.333	0.444	0.222
3	0.0	0.333	0.667	0.000

- State 1 has a 100% chance of moving to state 3 in the second month.
- State 2 has a 12.5% chance of staying in state 2, a 50% chance of moving to state 2, and a 37.5% chance of moving to state 3.
- State 3 has a 40% chance of moving to state 2 and a 60% chance of staying in state 3.

These matrices are useful for predicting the system's future states and understanding how the system evolves based on the initial state.

4.5 Computing Steady-State Probabilities

Table 4 – Steady-State Probabilities for First Month.

States	Probability
0	0.0
1	0.383
2	0.313
3	0.304

Table 5 – Steady-State Probabilities for Second Month.

States	Probability
0	0.0
1	0.0
2	0.267
3	0.733

1. **First Month :** The probabilities show that the queue system is relatively balanced, with the highest probability (38.3%) in state 1 and nearly equal probabilities in states 2 and 3. This suggests moderate queue lengths with effective management.

Table 6 – Steady-State Probabilities for Second Month.

States	Probability
0	0.0
1	0.231
2	0.385
3	0.384

2. **Second Month :** The probabilities indicate a high likelihood (73.3%) of the system being in state 3, showing significant inefficiencies and long queues. This month reflects the need for urgent improvements in queue management.
3. **Third Month :** The steady-state probabilities suggest a more even distribution across states $1, 2$, and 3, with state 2 having the highest probability (38.5%). This indicates improved queue management and resource allocation compared to the second month.

The steady-state analysis using the Markov chain model provides a comprehensive understanding of the queue dynamics over three months. The results highlight the importance of effective queue management strategies and optimal resource allocation to minimize customer waiting times and improve service efficiency in supermarkets. Future efforts should focus on maintaining and enhancing these strategies to ensure continued customer satisfaction and operational efficiency.

Table 7 – Arrival Time.

Month	Total Arrival Time (Arrivals/Hour)
First	Month 17.2
Second	Month 26.3
Third	Month 26.1

1. **First Month** : The total arrival time is 17.2 arrivals/hour, indicating a relatively moderate customer inflow. This aligns with the balanced steady-state probabilities observed.
2. **Second Month** : The total arrival time increases significantly to 26.3 arrivals/hour. This higher rate correlates with inefficient queue management, where the system frequently transitions to longer queues.
3. **Third Month** : The total arrival time is slightly higher at 26.1 arrivals/hour, reflecting the system's improved capacity to handle customer arrivals more effectively despite a high inflow. These results underscore the importance of understanding and managing arrival rates to optimize queue management and service efficiency in supermarkets. The steady-state probabilities, coupled with accurate arrival rates, provide crucial insights for operational improvements.

1. **First Month** : High utilization (0.93) indicates the system is almost fully occupied, leading to longer waiting times and more customers in the system and queue. This suggests potential inefficiencies and a need for improved service rates or additional servers.
2. **Second Month** : Slightly lower utilization (0.843) compared to the first month, leading to a reduction in waiting times and customers in the system and queue. This indicates an improvement in managing the queue.
3. **Third Month :** The lowest utilization (0.746) among the three months, resulting in the least number of customers in the system and queue and the shortest waiting times. This reflects effective queue management and optimal service rates.

These results highlight the importance of maintaining a balance between arrival and service rates to ensure efficient queue management and minimize customer wait times.

5 Conclusion

The findings of this study provide critical insights that can help supermarkets enhance their queue management strategies and improve operational efficiency. Through the application of the M/M/1 queuing model, the study was able to analyze arrival and service rates, as well as other key variables, to gain a deeper understanding of the factors driving customer wait times. This analysis revealed opportunities for supermarkets to optimize their resource allocation and queue management practices. By implementing more effective queue management strategies, supermarkets can significantly reduce customer waiting times and improve overall service efficiency. This, in turn, can lead to enhanced customer satisfaction and loyalty, as well as cost savings through more efficient resource utilization. The insights from this study can serve as a valuable guide for supermarket operators as they strive to strike the right balance between customer experience and operational cost-effectiveness. Continued research and application of queuing theory can further refine and enhance customer management practices in the supermarket industry.

Conflict of Interest

The authors have declared that there is no conflict of interest regarding this paper.

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Table 8 – Computations for Each Month.

Metric	First Month	Second Month	Third Month
Arrival Rate (λ) (arrivals/hour)	17.2	26.3	26.1
Service Rate (μ) (services/hour)	20	30	35
Utilization Factor (ρ)	0.93	0.843	0.746
Average No. of Customers in System (L)	9.11	4.46	3.07
Average No. of Customers in Queue (Lq)	9.20	3.76	2.29
Avg. Time Customer Spends in System (W)	0.556 hours	0.197 hours	0.113 hours
Avg. Time Customer Spends Waiting (Wq)	0.506 hours	0.149 hours	0.085 hours

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